

FINAL EXAM (TRANG) - ANSWER KEY

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Multiple Choice:

- (1) B (f_1, f_2, f_3 have to be solutions to a differential equation)
- (2) A
- (3) A
- (4) C
- (5) C
- (6) A
- (7) B
- (8) A
- (9) C (I think there's a typo in this question, the 1 should be an x)
- (10) A
- (11) A
- (12) B
- (13)

- (13.1) C (namely, $Proj(\mathbf{b})$ on $Col(A)$, but notice that any vector in $Col(A)$ has its second entry equal to 0)
- (13.2) B
- (14) A
- (15) A
- (16)

- (16.1) B
- (16.2) B
- (17) B (again, the 2 should be a $2x$)
- (18) A
- (19) A
- (20) A
- (21) A
- (22) B (I got $\begin{bmatrix} 0 \\ e^t + 4te^t \\ e^t + 2te^t \end{bmatrix}$)
- (23) B
- (24) A
- (25) A

(26) Both methods should give you the same answer, namely:

$$y(t) = \frac{7}{3}e^t - \frac{10}{3}e^{2t} - (t-1)e^{2t} + \frac{t^2}{2}e^{2t}$$

(27)

$$\mathbf{x}(t) = \begin{bmatrix} e^{2t} \\ 0 \end{bmatrix}$$

(use generalized eigenvectors, then undetermined coefficients. Then, if you plug in $t = 0$ in your general solution, you should get that $A = B = 0$)

(28)

$$[T]_{\mathcal{C} \leftarrow \mathcal{B}} = \begin{bmatrix} 0 & -1 & -2 \\ 0 & 1 & 3 \\ 1 & 1 & 0 \end{bmatrix}, \quad [T(1+t+t^2)]_{\mathcal{C}} = \begin{bmatrix} -3 \\ 4 \\ 0 \end{bmatrix}$$

(29)

$$\hat{\mathbf{v}} = \begin{bmatrix} -\frac{1}{2} \\ 0 \\ \frac{1}{2} \end{bmatrix}, \quad d = \sqrt{\frac{11}{2}}$$

Hint: The plane is spanned by $\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$